Fonctions polynomiales

Exercice 1

a) $p(x) = x^3 - 3x^2 + 4 \qquad d(x) = x^2 - x - 2$

$$\begin{array}{r|l} x^3 - 3x^2 + 0x + 4 & x^2 - x - 2 \\ \cline{2-2} -\,x^3 + x^2 + 2x & x - 2 \\ \cline{1-1} \quad -2x^2 + 2x + 4 & \\ \quad 2x^2 - 2x - 4 & \\ \cline{1-1} \quad\quad\quad 0 & \end{array}$$

$\Rightarrow \boxed{x^3 - 3x^2 + 4 = (x^2 - x - 2)(x - 2)}$

b) $p(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 19 \qquad d(x) = x^3 - 2x^2 - x + 2$

$$\begin{array}{r|l} x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 19 & x^3 - 2x^2 - x + 2 \\ \cline{2-2} -\,x^5 + 2x^4 + x^3 - 2x^2 & x^2 + 5x + 6 \\ \cline{1-1} \quad 5x^4 - 4x^3 - 17x^2 + 4x & \\ \quad -5x^4 + 10x^3 + 5x^2 - 10x & \\ \cline{1-1} \quad\quad 6x^3 - 12x^2 - 6x + 19 & \\ \quad\quad -6x^3 + 12x^2 + 6x - 12 & \\ \cline{1-1} \quad\quad\quad\quad 7 & \end{array}$$

$\Rightarrow \boxed{x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 18 = (x^3 - 2x^2 - x + 2)(x^2 + 5x + 6) + 7}$

c) $p(x) = 9x^4 + 9x^3 - 7x^2 - 10x + 5 \qquad d(x) = 3x^2 + 4x + 1$

$$\begin{array}{r|l} 9x^4 + 9x^3 - 7x^2 - 10x + 5 & 3x^2 + 4x + 1 \\ \cline{2-2} -\,9x^4 - 12x^3 - 3x^2 & 3x^2 - x - 2 \\ \cline{1-1} \quad -3x^3 - 10x^2 - 10x & \\ \quad 3x^3 + 4x^2 + x & \\ \cline{1-1} \quad\quad -6x^2 - 9x + 5 & \\ \quad\quad 6x^2 + 8x + 2 & \\ \cline{1-1} \quad\quad\quad -x + 7 & \end{array}$$

$\Rightarrow \boxed{9x^4 + 9x^3 - 7x^2 - 10x + 5 = (3x^2 + 4x + 1)(3x^2 - x - 2) - x + 7}$

d) $p(x) = x^3 + 2x^2 + 3x - 5 \qquad d(x) = x^2 - 2x - 1$

$$\begin{array}{r|l} x^3 + 2x^2 + 3x - 5 & x^2 - 2x - 1 \\ \cline{2-2} -\,x^3 + 2x^2 + x & x + 4 \\ \cline{1-1} \quad 4x^2 + 4x - 5 & \\ \quad -4x^2 + 8x + 4 & \\ \cline{1-1} \quad\quad 12x - 1 & \end{array}$$

$\Rightarrow \boxed{x^3 + 2x^2 + 3x - 5 = (x^2 - 2x - 1)(x + 4) + 12x - 1}$

Exercice 2

a) $x^3 - 9x^2 + 23x - 15 = 0$

$x^3 - 9x^2 + 23x - 15$ divisible par $(x-1)$

	1	-9	23	-15
$\cdot 1$		1	-8	15
	1	-8	15	0

$(x-1)(x^2 - 8x + 15) = 0$

$\Rightarrow (x-1)(x-3)(x-5) = 0 \qquad \Rightarrow \boxed{S = \{1; 3; 5\}}$

b) $x^3 + 5x^2 - 8x - 48 = 0$

$x^3 + 5x^2 - 8x - 48$ divisible par $(x-3)$

	1	5	-8	-48
$\cdot 3$		3	24	48
	1	8	16	0

$(x-3)(x^2 + 8x + 16) = 0$

$\Rightarrow (x-3)(x+4)^2 = 0 \qquad \Rightarrow \boxed{S = \{-4; 3\}}$

c) $x^3 + 3x^2 - 16x - 48 = 0$

$x^2(x+3) - 16(x+3) = 0$

$(x+3)(x^2 - 16) = 0$

$\Rightarrow (x+3)(x-4)(x+4) = 0 \qquad \Rightarrow \boxed{S = \{-4; -3; 4\}}$

d) $x^4 + 2x^3 - 4x^2 - 5x - 6 = 0$

$x^4 + 2x^3 - 4x^2 - 5x - 6$ divisible par $(x-2)$

	1	2	-4	-5	-6
$\cdot 2$		2	8	8	6
	1	4	4	3	0

$(x-2)(x^3 + 4x^2 + 4x + 3) = 0$

x^3+4x^2+4x+3 divisible par $(x+3)$

1	4	4	3
	-3	-3	-3
1	1	1	0

$(x+3)(x^2+x+1)=0$ (x^2+x+1 ne se factorise pas $\Delta=-3<0$)

$\Rightarrow (x-2)(x+3)(x^2+x+1)=0$ $\Rightarrow \boxed{S=\{-3;2\}}$

e) $35x^3+47x^2+13x+1=0$

$35x^3+47x^2+13x+1$ divisible par $(x+1)$

35	47	13	1
	-35	-12	-1
35	12	1	0

$(x+1)(35x^2+12x+1)=0$

$\Rightarrow (x+1)(7x+1)(5x+1)=0$ $\Rightarrow \boxed{S=\left\{-1;-\frac{1}{5};-\frac{1}{7}\right\}}$

f) $6x^3-17x^2+14x-3=0$

$6x^3-17x^2+14x-3$ divisible par $(x-1)$

6	-17	14	-3
	6	-11	3
6	-11	3	0

$(x-1)(6x^2-11x+3)=0$

$\Rightarrow (x-1)(2x-3)(3x-1)=0$ $\Rightarrow \boxed{S=\left\{\frac{1}{3};1;\frac{3}{2}\right\}}$

Exercice 3

a) $x^3 - 2x^2 - 5x + 6 > 0$

$x^3 - 2x^2 - 5x + 6$ divisible par $(x - 1)$

1	-2	-5	6
	1	-1	-6
1	-1	-6	0

$(x - 1)(x^2 - x - 6) > 0$

$\Rightarrow (x - 1)(x - 3)(x + 2) > 0$ $\Rightarrow \boxed{S =]-2; 1[\cup]3; +\infty[}$

x	$-\infty$		-2		1		3		$+\infty$
$x-1$		$-$		$-$	0	$+$		$+$	
$x-3$		$-$		$-$		$-$	0	$+$	
$x+2$		$-$	0	$+$		$+$		$+$	
$f(x)$		$-$	0	$+$	0	$-$	0	$+$	

b) $x^3 - 8x^2 - 9x + 72 < 0$

$x^3 - 8x^2 - 9x + 72$ divisible par $(x - 3)$

1	-8	-9	72
	3	-15	-72
1	-5	-24	0

$(x - 3)(x^2 - 5x - 24) < 0$

$\Rightarrow (x - 3)(x - 8)(x + 3) < 0$ $\Rightarrow \boxed{S =]-\infty; -3[\cup]3; 8[}$

x	$-\infty$		-3		3		8		$+\infty$
$x-3$		$-$		$-$	0	$+$		$+$	
$x-8$		$-$		$-$		$-$	0	$+$	
$x+3$		$-$	0	$+$		$+$		$+$	
$f(x)$		$-$	0	$+$	0	$-$	0	$+$	

c) $4(x^4+1) \geq 17x^2$ $\qquad 4x^4 - 17x^2 + 4 \geq 0$

$(4x^2-1)(x^2-4) \geq 0$ $\qquad (2x-1)(2x+1)(x-2)(x+2) \geq 0$

x	$-\infty$		-2		$-\frac{1}{2}$		$\frac{1}{2}$		2		$+\infty$
$2x-1$	$-$		$-$		$-$	0	$+$		$+$		
$2x+1$	$-$		$-$	0	$+$		$+$		$+$		
$x-2$	$-$		$-$		$-$		$-$	0	$+$		
$x+2$	$-$	0	$+$		$+$		$+$		$+$		
$f(x)$	$+$	0	$-$	0	$+$	0	$-$	0	$+$		

$\Rightarrow$ $\boxed{S =]-\infty; -2] \cup [-\frac{1}{2}; \frac{1}{2}] \cup [2; \infty[}$

d) $6x^3 - 8x^2 \leq 6x - 8$ $\qquad 6x^3 - 8x^2 - 6x + 8 \leq 0$

$2x^2(3x-4) - 2(3x-4) \leq 0$ $\qquad (3x-4)(2x^2-2) \leq 0$

$2(3x-4)(x^2-1) \leq 0$ $\qquad 2(3x-4)(x-1)(x+1)) \leq 0$

x	$-\infty$		-1		1		$\frac{4}{3}$		$+\infty$
$2(3x-4)$	$-$		$-$		$-$	0	$+$		
$x-1$	$-$		$-$	0	$+$		$+$		
$x+1$	$-$	0	$+$		$+$		$+$		
$f(x)$	$-$	0	$+$	0	$-$	0	$+$		

$\Rightarrow$ $\boxed{S =]-\infty; -1] \cup [1; \frac{4}{3}]}$

Exercice 4

a) $a(x) = x^3 - 8x^2 + x + 42$ divisible par $(x-3)$

1	-8	1	42
	3	-15	-42
1	-5	-14	0

$a(x) = (x-3)(x^2 - 5x - 14) = 0$

$\Rightarrow a(x) = (x-3)(x+2)(x-7)$

x	$-\infty$		-2		3		7		$+\infty$
$x-3$	$-$		$-$		0	$+$		$+$	
$x+2$	$-$	0	$+$		$+$			$+$	
$x-7$	$-$		$-$		$-$		0	$+$	
$a(x)$	$-$	0	$+$	0	$-$	0		$+$	

b) $b(x) = -(x^3 - 10x^2 + 31x - 30)$ divisible par $(x-2)$

1	-10	31	-30
	2	-16	30
1	-8	15	0

$b(x) = -(x-2)(x^2 - 8x + 15) = 0$

$\Rightarrow b(x) = -(x-2)(x-3)(x-5)$

x	$-\infty$		2		3		5		$+\infty$
$-(x-2)$	$+$	0	$-$		$-$			$-$	
$x-3$	$-$		$-$	0	$+$			$+$	
$x-5$	$-$		$-$		$-$	0		$+$	
$b(x)$	$+$	0	$-$	0	$+$	0		$-$	

c) $c(x) = -(4x^3 - 7x^2 - 5x + 6)$ divisible par $(x+1)$

4	-7	-5	6
	-4	11	-6
4	-11	6	0

$c(x) = -(x+1)(4x^2 - 11x + 6) = 0$

$\Rightarrow c(x) = -(x+1)(4x-3)(x-2)$

x	$-\infty$		-1		$\frac{3}{4}$		2	$+\infty$
$-(x+1)$		$+$	0	$-$		$-$		$-$
$4x-3$		$-$		$-$	0	$+$		$+$
$x-2$		$-$		$-$		$-$	0	$+$
$c(x)$		$+$	0	$-$	0	$+$	0	$-$

d) $d(x) = 6x^3 + 5x^2 - 12x + 4$ divisible par $(x+2)$

6	5	-12	4
	-12	14	-4
6	-7	2	0

$d(x) = (x+2)(6x^2 - 7x + 2) = 0$

$\Rightarrow d(x) = (x+2)(2x-1)(3x-2)$

x	$-\infty$		-2		$\frac{1}{2}$		$\frac{2}{3}$	$+\infty$
$x+2$		$-$	0	$+$		$+$		$+$
$2x-1$		$-$		$-$	0	$+$		$+$
$3x-2$		$-$		$-$		$-$	0	$+$
$d(x)$		$-$	0	$+$	0	$-$	0	$+$

Exercice 5

$p(x) = a(x-1)(x-6)(x-2)$

$p(5) = 60$

$p(5) = a(4)(-1)(3) = 60$

$-12a = 60 \qquad \Rightarrow a = -5$

$p(x) = -5(x-1)(x-6)(x-2)$

$p(x) = -5(x-1)(x^2-8x+12)$

$p(x) = -5(x^3-9x^2+20x-12)$

$\boxed{p(x) = -5x^3+45x^2-100x+60}$

Exercice 6

$p(x) = (x-7)(x+2)(ax+b)$

$p(-3) = 55 \qquad\qquad p(2) = -10$

$p(-3) = (-10)(-1)(-3a+b) = 55 \qquad\qquad p(2) = (-5)(4)(2a+b) = -10$

$\begin{cases} 10(-3a+b) = 55 \\ -20(2a+b) = -10 \end{cases} \qquad \begin{cases} -30a+10b = 55 \\ -40a-20b = -10 \end{cases}$

$\begin{cases} -60a+20b = 110 \\ -40a-20b = -10 \end{cases} \qquad \Rightarrow -100a = 100$

$\Rightarrow a = -1 \qquad\qquad \Rightarrow b = \dfrac{55-30}{10} = \dfrac{5}{2}$

$p(x) = (x-7)(x+2)(-x+\frac{5}{2})$

$p(x) = (x^2-5x-14))(-x+\frac{5}{2})$

$\boxed{p(x) = -x^3+\dfrac{15}{2}x^2+\dfrac{3}{2}x-35}$

Exercice 7

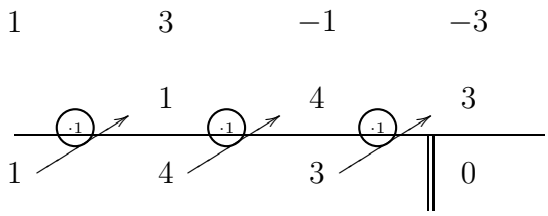
$x =$ le plus petit nombre nombre

$x(x+1)(x+2) = x + x + 1 + x + 2$

$x^3 + 3x^2 + 2x = 3x + 3$

$x^3 + 3x^2 - x - 3 = 0$

$x^3 + 3x^2 - x - 3$ est divisible par $(x-1)$



$(x-1)(x^2+4x+3) = 0$

$(x-1)(x+3)(x+1) = 0$

Il y a 3 solutions: $(1;2;3)$ ou $(-3;-2;-1)$ ou $(-1;0;1)$