Factorisation et division euclidienne

Exercice 1.

a) polynôme divisible par $(x-1)$

$$\begin{array}{c|cccc} & 1 & -4 & -7 & 10 \\ 1 & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & \boxed{0} \end{array} \quad \Rightarrow \quad (x-1)(x^2-3x-10) = \boxed{(x-1)(x-5)(x+2)}$$

b) polynôme divisible par $(x-2)$

$$\begin{array}{c|cccc} & 1 & 2 & -5 & -6 \\ 2 & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & \boxed{0} \end{array} \quad \Rightarrow \quad (x-2)(x^2+4x+3) = \boxed{(x-2)(x+3)(x+1)}$$

c) polynôme divisible par $(x-3)$

$$\begin{array}{c|cccc} & 2 & -3 & -8 & -3 \\ 3 & & 6 & 9 & 3 \\ \hline & 2 & 3 & 1 & \boxed{0} \end{array} \quad \Rightarrow \quad (x-3)(2x^2+3x+1) = \boxed{(x-3)(2x+1)(x+1)}$$

d) $x(x^2-9) = \boxed{x(x-3)(x+3)}$

e) polynôme divisible par $(x-1)$

$$\begin{array}{c|cccc} & 1 & -6 & 11 & -6 \\ 1 & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array} \quad \Rightarrow \quad (x-1)(x^2-5x+6) = \boxed{(x-1)(x-2)(x-3)}$$

f) $x^2(x^2+5x+6) = \boxed{x^2(x+2)(x+3)}$

g) $x(x^3+2x^2-x-2) = x[x^2(x+2)-1(x+2)] = x(x+2)(x^2-1) = \boxed{x(x+2)(x-1)(x+1)}$

h) $x(2x^3-3x^2-2x+3) = x[x^2(2x-3)-1(2x-3)] = x(2x-3)(x^2-1) = \boxed{x(2x-3)(x-1)(x+1)}$

i) $(x^2-1)(x^2-3) = \boxed{(x-1)(x+1)(x-\sqrt{3})(x+\sqrt{3})}$

j) $x^2(x+1)-4(x+1) = (x+1)(x^2-4) = \boxed{(x+1)(x-2)(x+2)}$

k) $3x(x^2-4) = \boxed{3x(x-2)(x+2)}$

l) $x^3(x-3)-1(x-3) = (x-3)(x^3-1) = \boxed{(x-3)(x-1)(x^2+x+1)}$

m) $x^2(x-2)-1(x-2)=(x-2)(x^2-1)=\boxed{(x-2)(x-1)(x+1)}$

n) $x^2(x+3)-4(x+3)=(x+3)(x^2-4)=\boxed{(x+3)(x-2)(x+2)}$

Exercice 2.

a) $x^3-9x^2+23x-15$ divisible par $(x-1)$

	1	-9	23	-15
1		1	-8	15
	1	-8	15	0

$\Rightarrow (x-1)(x^2-8x+15)=(x-1)(x-3)(x-5)$

$\Rightarrow \boxed{\mathcal{S}=\{1;3;5\}}$

b) $x^3+5x^2-8x-48$ divisible par $(x-3)$

	1	5	-8	-48
3		3	24	48
	1	8	16	0

$\Rightarrow (x-3)(x^2+8x+16)=(x-3)(x+4)^2$

$\Rightarrow \boxed{S=\{-4;3\}}$

c) $x^3+3x^2-16x-48=0 \Leftrightarrow x^2(x+3)-16(x+3)=0 \Leftrightarrow (x+3)(x^2-16)=0$

$\Leftrightarrow (x+3)(x-4)(x+4)=0 \Rightarrow \boxed{S=\{-4;-3;4\}}$

d) $x^4+2x^3-4x^2-5x-6$ divisible par $(x-2)$

	1	2	-4	-5	-6
2		2	8	8	6
	1	4	4	3	0

$\Rightarrow (x-2)(x^3+4x^2+4x+3)$

x^3+4x^2+4x+3 divisible par $(x+3)$

	1	4	4	3
-3		-3	-3	-3
	1	1	1	0

$\Rightarrow (x+3)(x^2+x+1)$

(x^2+x+1 ne se factorise pas $\Delta=-3<0$)

$\Rightarrow (x-2)(x+3)(x^2+x+1)=0 \quad \Rightarrow \boxed{S=\{-3;2\}}$

e) $35x^3+47x^2+13x+1$ divisible par $(x+1)$

$$\begin{array}{r|cccc} & 35 & 47 & 13 & 1 \\ -1 & & -35 & -12 & -1 \\ \hline & 35 & 12 & 1 & \boxed{0} \end{array} \quad \Rightarrow \quad (x+1)(35x^2+12x+1) = (x+1)(7x+1)(5x+1)$$

$$\Rightarrow (x+1)(7x+1)(5x+1) = 0 \qquad \Rightarrow \boxed{S = \left\{-1; -\frac{1}{5}; -\frac{1}{7}\right\}}$$

f) $6x^3 - 17x^2 + 14x - 3$ divisible par $(x-1)$

$$\begin{array}{r|cccc} & 6 & -17 & 14 & -3 \\ 1 & & 6 & -11 & 3 \\ \hline & 6 & -11 & 3 & \boxed{0} \end{array} \quad \Rightarrow \quad (x-1)(6x^2-11x+3) = (x-1)(2x-3)(3x-1)$$

$$\Rightarrow (x-1)(2x-3)(3x-1) = 0 \qquad \Rightarrow \boxed{S = \left\{\frac{1}{3}; 1; \frac{3}{2}\right\}}$$

Exercice 3.

$p(x) = a(x-1)(x-6)(x-2)$

$p(5) = 60 \quad \Leftrightarrow \quad a(4)(-1)(3) = 60 \quad \Leftrightarrow \quad -12a = 60 \quad \Leftrightarrow \quad a = -5$

$p(x) = -5(x-1)(x-6)(x-2)$

$p(x) = -5(x-1)(x^2-8x+12)$

$p(x) = -5(x^3-9x^2+20x-12)$

$\boxed{p(x) = -5x^3+45x^2-100x+60}$

Exercice 4.

$p(x) = (x-7)(x+2)(ax+b)$

$p(-3) = 55 \qquad\qquad p(2) = -10$

$p(-3) = (-10)(-1)(-3a+b) = 55 \qquad\qquad p(2) = (-5)(4)(2a+b) = -10$

$$\begin{cases} 10(-3a+b) = 55 \\ -20(2a+b) = -10 \end{cases} \qquad \begin{cases} -30a+10b = 55 \\ -40a-20b = -10 \end{cases}$$

$$\begin{cases} -60a+20b = 110 \\ -40a-20b = -10 \end{cases} \qquad \Rightarrow -100a = 100$$

$\Rightarrow a = -1 \qquad\qquad \Rightarrow b = \dfrac{55-30}{10} = \dfrac{5}{2}$

$p(x) = (x-7)(x+2)(-x+\frac{5}{2})$

$p(x) = (x^2-5x-14))(-x+\frac{5}{2})$

$\boxed{p(x) = -x^3+\frac{15}{2}x^2+\frac{3}{2}x-35}$