Trigonométrie

Exercice 1.

Théorème de Pythagore : $c = \sqrt{6^2 + 9^2} = \sqrt{117} = 3\sqrt{13} \simeq \boxed{10{,}82 \text{ cm}}$

$\tan(\alpha) = \frac{6}{9} \quad \Leftrightarrow \quad \alpha = \arctan\left(\frac{6}{9}\right) \simeq \boxed{33{,}69°} \qquad \beta = 90 - \alpha \simeq \boxed{56{,}31°}$

aire du $\Delta\ ABC = \frac{6 \cdot 9}{2} = \boxed{27 \text{ cm}^2}$

Exercice 2.

$\gamma = 90° - \alpha = \boxed{55°}$

$\sin(35°) = \frac{5}{b} \quad \Leftrightarrow \quad b = \frac{5}{\sin(35°)} \simeq \boxed{8{,}72 \text{ cm}}$

$\tan(35°) = \frac{5}{c} \quad \Leftrightarrow \quad c = \frac{5}{\tan(35°)} \simeq \boxed{7{,}14 \text{ cm}}$

aire du $\Delta\ ABC = \frac{5 \cdot c}{2} \simeq \boxed{17{,}85 \text{ cm}^2}$

Exercice 3.

théorème du cosinus : $\alpha = \arccos\left(\frac{9^2 + 4^2 - 6^2}{2 \cdot 9 \cdot 4}\right) \cong \boxed{32{,}09°}$

théorème du sinus : $\frac{6}{\sin(\alpha)} = \frac{9}{\sin(\beta)} = \frac{4}{\underbrace{\sin(\gamma)}_{\text{angle aigu}}}$

$\gamma = \arcsin\left(\frac{4 \cdot \sin(\alpha)}{6}\right) \cong \boxed{20{,}74°}$

$\beta = 180° - \alpha - \gamma \cong \boxed{127{,}17°}$

$S = \sigma(\Delta\ ABC) = \frac{1}{2} \cdot 9 \cdot 4 \cdot \sin(\alpha) \cong \boxed{9{,}56\ u^2}$

théorème du sinus : $\frac{5}{\sin(35°)} = \frac{7}{\underbrace{\sin(\beta)}_{\text{angle aigu ou obtus}}} = \frac{c}{\sin(\gamma)}$

$\beta_1 = \arcsin\left(\frac{7 \cdot \sin(35°)}{5}\right) \cong \boxed{53{,}42°}$

$\gamma_1 = 180° - 35° - \beta_1 \cong \boxed{91{,}58°}$

$c_1 = \frac{5 \cdot \sin(\gamma_1)}{\sin(35°)} \cong \boxed{8{,}71\ u}$

$S_1 = \sigma(\Delta ABC) = \frac{1}{2} \cdot 5 \cdot 7 \cdot \sin(\gamma_1) \cong \boxed{17{,}49\ u^2}$

$\beta_2 = 180° - \beta_1 \cong \boxed{126{,}58°}$

$\gamma_2 = 180° - 35° - \beta_2 \cong \boxed{18{,}42°}$

$c_2 = \frac{5 \cdot \sin(\gamma_2)}{\sin(35°)} \cong \boxed{2{,}75\ u}$

$S_2 = \sigma(\Delta ABC) = \frac{1}{2} \cdot 5 \cdot 7 \cdot \sin(\gamma_2) \cong \boxed{5{,}53\ u^2}$

Exercice 4.

$\sphericalangle\, ABS = 180° - 60° = 120° \quad \Rightarrow \quad \sphericalangle ASB = 180° - 40° - 120° = 20°$

théorème du sinus dans le triangle ABS : $\dfrac{300}{\sin(20°)} = \dfrac{BS}{\sin(40°)}$

$$BS = \frac{300 \cdot \sin(40°)}{\sin(20°)} \cong 563{,}82 \text{ m}$$

$h = BS \cdot \sin(60°) \cong 488{,}28 \text{ m}$

altitude du sommet S : $1250 + h \cong \boxed{1738{,}28 \text{ m}}$