Exponentielles et logarithmes

Exercice 1

a) $7^{x+3} = 7^2 \quad \Rightarrow \quad x+3=2 \quad \Rightarrow \quad \boxed{S=\{-1\}}$

b) $2^{3x-5} = 2^{4x} \quad \Rightarrow \quad 3x-5=4x \quad \Rightarrow \quad \boxed{S=\{-5\}}$

c) $5^{-x+4} = 250 \quad \Rightarrow \quad -x+4=\log_5(250) \quad \Rightarrow \quad \boxed{S=\{4-\log_5(250)\}}$

d) $\log_x(100) = 2 \quad \Rightarrow \quad x^2=100 \quad \Rightarrow \quad \boxed{S=\{10\}}$ (-10 sol. à élim.)

e) $\log_{3x}(216) = 3 \quad \Rightarrow \quad (3x)^3=216 \quad \Rightarrow \quad 3x=6 \quad \Rightarrow \quad \boxed{S=\{2\}}$

f) $\log\left(\frac{4x-1}{3}\right) = 3 \quad \Rightarrow \quad \frac{4x-1}{3}=10^3 \quad \Rightarrow \quad x=\frac{3001}{4} \quad \Rightarrow \quad \boxed{S=\left\{\frac{3001}{4}\right\}}$

g) $\log_a(x) = \log_a\left(\frac{12^2}{2^3\cdot 6}\right) \quad \Rightarrow \quad x=\frac{144}{48}=3 \quad \Rightarrow \quad \boxed{S=\{3\}}$

Exercice 2

a) $P(t) = 12,3\cdot\left(\frac{12,3}{7,2}\right)^{\frac{t}{10}} = \boxed{12,3\cdot\left(\frac{41}{24}\right)^{\frac{t}{10}}}$

b) $P(10) = 12,3\cdot\left(\frac{41}{24}\right) \cong \boxed{21,0125 \text{ milliards}}$

c) $P(t) = P_0\cdot\left(\frac{41}{24}\right)^{\frac{t}{10}} = 2\cdot P_0 \quad \Rightarrow \quad \left(\frac{41}{24}\right)^{\frac{t}{10}}=2$

$\frac{t}{10} = \log_{(41/24)}(2) \quad \Rightarrow \quad t = 10\cdot\log_{(41/24)}(2) \cong 12,94 \text{ années} \quad \Rightarrow \quad \boxed{13 \text{ années}}$

Exercice 3

a) $\log_8(8^2) = \boxed{2}$ b) $\log_{11}\left(11^{-3}\right) = \boxed{-3}$ c) $\log_a\left(a^{\frac{2}{3}}\right) = \boxed{\frac{2}{3}}$

Exercice 4

a) $-5x - 17 > 0 \quad \Rightarrow \quad x < -\dfrac{17}{5} \quad \Rightarrow \boxed{ED(f) =]-\infty;\ -\tfrac{17}{5}[}$

b) $-x^2 + x + 20 > 0 \quad \Rightarrow \quad x^2 - x - 20 < 0 \quad \Rightarrow (x-5)(x+4) < 0$

$\Rightarrow \boxed{ED(g) =]-4; 5[}$

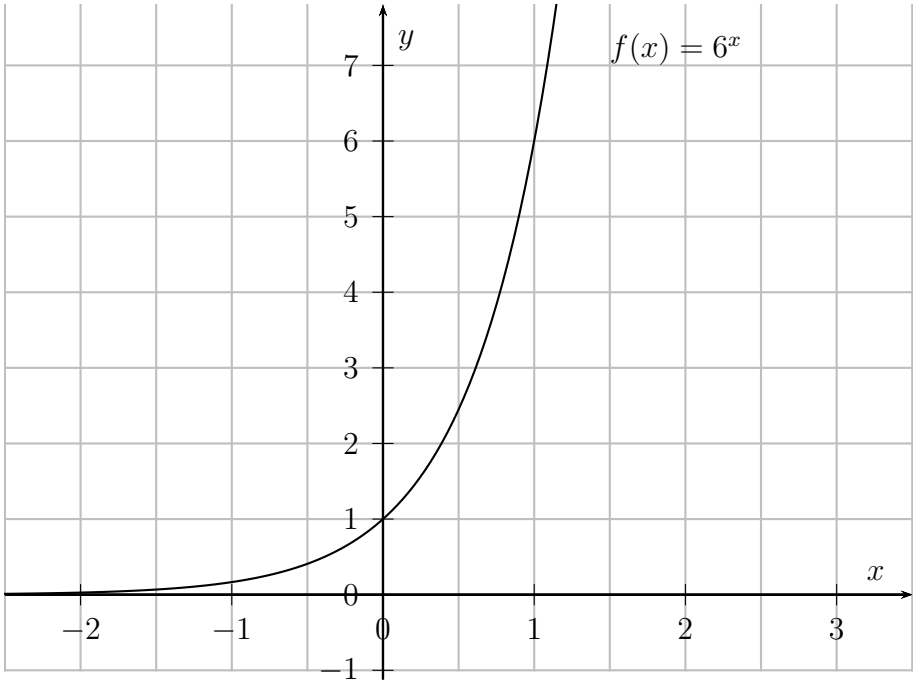
c) $18 - 3^{2x-1} = 0 \quad \Rightarrow \quad \log_3(18) = 2x - 1 \quad \Rightarrow \ 2x = \log_3(9 \cdot 2) + 1$

$\Rightarrow \ 2x = 2 + \log_3(2) + 1 \quad \Rightarrow \ x = \dfrac{3 + \log_3(2)}{2} \cong 1,815 \quad \Rightarrow \boxed{ED(h) = \mathbb{R} - \left\{\tfrac{3+\log_3(2)}{2}\right\}}$

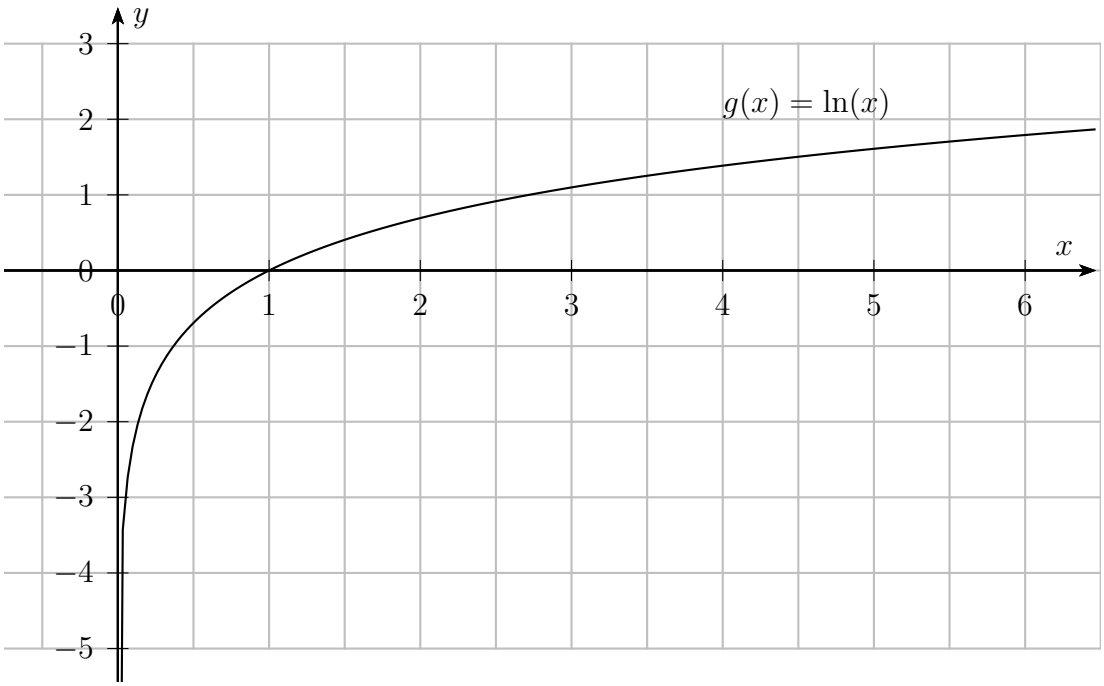
d) $\dfrac{x^2 - 36}{x - 5} > 0 \quad \Rightarrow \quad \dfrac{(x-6)(x+6)}{x-5} > 0 \quad \Rightarrow \boxed{ED(i) =]-6; 5[\cup]6; +\infty[}$

Exercice 5

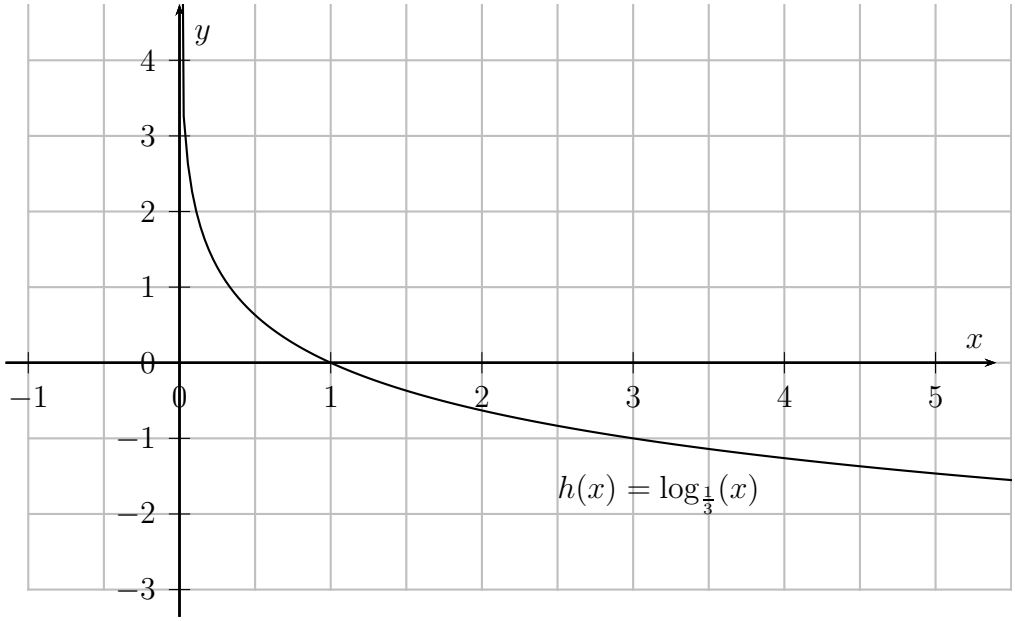
a) $f(x) = 6^x \qquad ED(f) = \mathbb{R}$



b) $g(x) = \ln(x)$ $\quad ED(g) = \mathbb{R}_+^*$



c) $h(x) = \log_{\frac{1}{3}}(x)$ $\quad ED(h) = \mathbb{R}_+^*$



Exercice 6

$C_o = 4'720$ CHF $\quad t = 3,5 \quad n = 12$ ans

$$C_n = C_o \cdot \left(1 + \frac{t}{100}\right)^n \qquad \Rightarrow C_{12} = 4'720 \cdot (1,035)^{12} \cong \boxed{7'132,25 \text{ CHF}}$$

Exercice 7

$C_5 = 5'039,20$ CHF $\qquad t = 2,75 \qquad n = 5$ ans

$C_n = C_o \cdot \left(1 + \dfrac{t}{100}\right)^n \qquad \Rightarrow C_5 = 5'039,20 = C_o \cdot 1,0275^5$

$\Rightarrow \; C_0 = \dfrac{5'039,20}{1,0275^5} \cong \boxed{4'400 \text{ CHF}}$

Exercice 8

$C_0 = 3'240$ CHF $\qquad C_{18} = 6'563,65$ CHF $\qquad n = 18$ ans

$C_n = C_o \cdot \left(1 + \dfrac{t}{100}\right)^n \qquad \Rightarrow C_{18} = 6'563,65 = 3'240 \cdot \left(1 + \dfrac{t}{100}\right)^{18}$

$\Rightarrow \; \dfrac{t}{100} = \sqrt[18]{\dfrac{6'563,65}{3'240}} - 1 \cong 0,04 \qquad \Rightarrow \boxed{t = 4} \Rightarrow$ le taux est de 4 %

Exercice 9

$C_0 = 790$ CHF $\qquad C_n = 10'000$ CHF $\qquad t = 3,5$

$C_n = C_o \cdot \left(1 + \dfrac{t}{100}\right)^n \qquad \Rightarrow C_n = 10'000 = 790 \cdot 1,035^n$

$\Rightarrow \; 1,035^n = \dfrac{10'000}{790} = \dfrac{1000}{79} \qquad \Rightarrow \; n = \log_{1,035}\left(\dfrac{1000}{79}\right) \cong 73,78 \quad \Rightarrow \boxed{74 \text{ années}}$

Exercice 10

$m(t) = m_o \cdot 0,5^{t/5568}$

a) $0,05 \cdot m_o = m_o \cdot 0,5^{t/5568} \qquad \Rightarrow 0,5^{t/5568} = 0,05$

$\Rightarrow \; \frac{t}{5568} = \log_{0,5}(0,05) \qquad \Rightarrow t = 5568 \cdot \log_{0,5}(0,05) \cong \boxed{24'064 \text{ années}}$

b) $0,528 \cdot m_o = m_o \cdot 0,5^{t/5568} \qquad \Rightarrow 0,5^{t/5568} = 0,528$

$\Rightarrow \; \frac{t}{5568} = \log_{0,5}(0,528) \qquad \Rightarrow t = 5568 \cdot \log_{0,5}(0,528) \cong \boxed{5'130 \text{ années}}$

Exercice 11

a) $\boxed{f(t) = 8 \cdot 3^{\frac{t}{5}}}$

b) $8 \cdot 3^{\frac{t}{5}} = 10^6 \qquad \Rightarrow \; \dfrac{t}{5} = \log_3\left(\dfrac{10^6}{8}\right) = \log_3(125'000) \cong 10,683$

$\Rightarrow t \cong 53,413$ min $\qquad \Rightarrow \boxed{t \cong 53 \text{ min } 25 \text{ sec}}$